Spin - Orbit Resonances of High - Eccentricity Asteroids and Other Minor Planets (e > 0.9)

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- ## Motivation

Some asteroids in the Solar system have eccentricities above 0.9. Their origin and destiny are not clear. They may be remnants of the primordial asteroid belt, results of relatively recent interaction with planets, or even captured objects. The most notable high-eccentricity object in the JPL Horizons database is 2006 HY51 with an eccentricity of 0.9684 https://ssd.jpl.nasa.gov/sbdb.cgi#top, but a few other objects with slightly smaller eccentricity have been detected. These Apollo class asteroids are Near-Earth Objects. Similar objects were involved in the bombardment of inner planets and accretion of primordial terrestrial planets. Around white dwarf planetary systems, highly eccentric asteroids are thought to be the primary progenitor of debris disks (Jura 2003; Debes et al. 2012; Veras et al. 2014; Malamud & Perets 2020a,b) and observed metallic pollution in the photospheres of host stars (Zuckerman et al. 2010; Koester et al. 2014).

We investigate the rotational states of high-eccentricity triaxial objects in the basic 1D model (i.e., regarding only the spin about the principal axis of inertia) on the example of 2006 HY51, the most eccentric asteroid on a closed orbit in the solar system.

• Relevant parameters and functions

```
oneAU = 1.496*^11;
Rsun = 6.957*^8;(* radius of Sun in m *)
Msun = 1.98892*^30;(* mass of Sun in kg *)
G = 6.67428*^-11; (* gravitational constant in m^3 kg^-1 s^-2  *)
R = 0.609*^3; (* radius of 2006 HY51 in m *) ;
M1 = 1.0 * Msun; (* mass of host star = Sun *)
M2 = 4. / 3 * Pi * R ^ 3 * 2. ^ 3;
  (* mass of 2006 HY51 in kg, a guess of mean density *) ;
bca = 0.2; (* (B-A)/C a placeholder, also called σ *)
a = 2.59 * oneAU; (* semimajor axis in m*)
n = 2 * Pi / 4.17 / 365.25; (* mean motion of asteroid  rad/day *) ;
e = 0.9684; (* eccentricity of 2006 HY51  *)
(* Mean anomaly function *)
M[t_, n_] := Mod[n * t, 2 Pi];
(* True anomaly function *)
Ta[E_, e_] := 2 ArcTan[Cos[E / 2.], Sqrt[(1 + e) / (1 - e)] Sin[E / 2.]];
```

• Direct integration of rotation process

It is straightforward to establish that rotation of high-e asteroids is intrinsically chaotic by direct integration of the well-known equation of motion (Wisdom, J., Peale, S. J., Mignard, F. 1984, Icarus, 58, 137; Makarov V.~V., Veras D., 2019, ApJ, 886, 127). If we start integration with random initial conditions, impulse-like interactions at perihelion change the spin rate $\omega == \dot{\theta} / n$ by apparently random increments resulting in a stochastic random process process.

In[∘]:=
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σ = bca; norb = 400; timespan = norb * 2 * Pi / n; e = 0.9684;
(* Eccentric anomaly function by inverse interpolation *)
EE = Interpolation[Table[{i - e * Sin[i], i}, {i, 0, 6.5, 0.0005}], Method → "Spline"];
solTID = NDSolve[{y''[t] + 3 / 2 * n^2 * σ / (1 - e Cos[EE[M[t, n]]]) ^3 *
      (Sin[2 * y[t] - 2 * Ta[EE[M[t, n]], e]]) == 0, y[0] == -0.04, y'[0] == 800. n},
   {y'[t], y[t]}, {t, 0, timespan}, MaxSteps → ∞, AccuracyGoal → 10,
   PrecisionGoal → 11, Method → "StiffnessSwitching"];
velo[t_] = y'[t] /. solTID;
angl[t_] = y[t] /. solTID;
per = 2 Pi / n; ListPlot[
 Table[{(i + 0.5), velo[(i + 0.5) * per] / n} // Flatten, {i, 0, norb - 1}], Frame → True,
 FrameLabel → {Style["time orbits", FontSize → 14], Style["θ̇/n", FontSize → 14]},
 ImageSize → 400, PlotRange → All]
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Out[∘]=

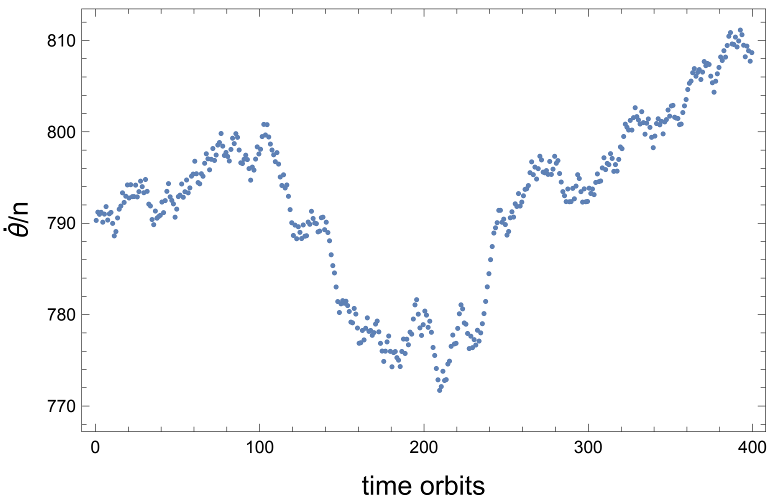

Fig.1 Simulation of relative spin rate of 2006 HY51 by direct integration of 400 orbits. Only aphelion values are shown.

The spin rate varies between approximately 12 n and 1100 n. It is limited on the low end but not on the high end.

Trial integrations over 9000 orbits show that the chaotic process is confined to a certain area in the $\{d_\omega, \omega\}$ space, where d_ω is the rotation velocity change between two consecutive aphelia.

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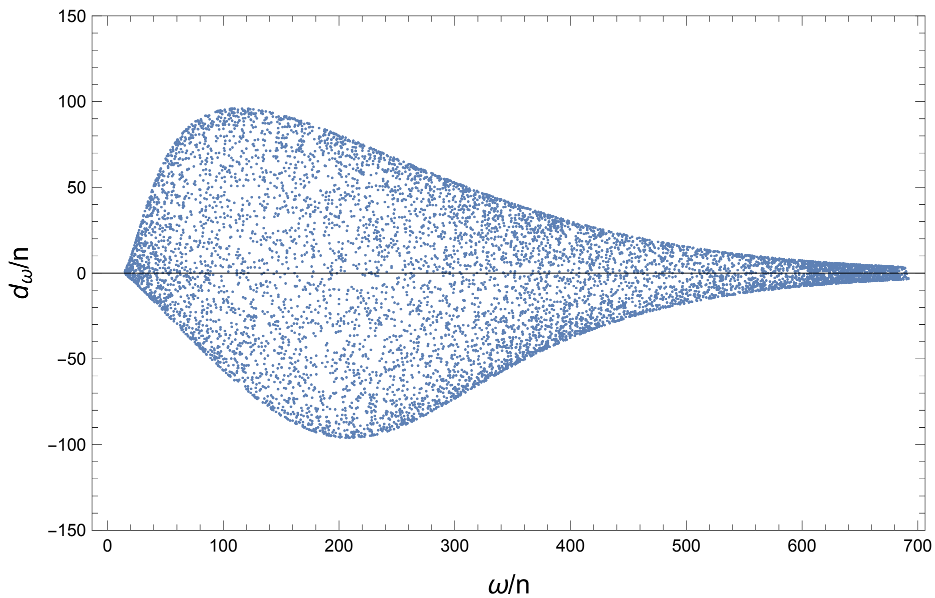

Fig.2 Spin rate updates versus aphelion spin rates of 2006 HY51 obtained by direct integration of 9000 orbits. Note the open-ended distribution at the high end.

The asymmetry of this distribution (Fig. 2) indicates a self-regulating process as positive spin updates are statistically larger at low velocities, and vice versa. Over a long time, the spin rate tends to wander in the mid range but it can be stuck in the "bottleneck" area of high spin for extended periods of time. Other trials showed that ω can reach as high as ~1100 n.

• Simulating chaotic rotation for gigayears

Rotational fission suggested in (Makarov V.~V., Veras D., 2019, ApJ, 886, 127; Veras, D., McDonald, C.~H., & Makarov, V.~V. 2020, \mnras, 492, 5291) for WD systems is not feasible for HY51 but it may be realized for higher eccentricity (Makarov, et al. 2020, accepted in ApJ, available here). To prove this, we had to develop a fast alternative to spin-orbit integration, which is quite slow because of the stiffness of the ODE and the required small time step. A Julia script is available at this https url via github. The basic idea is to replace the costly ODE integration with generation of $\{d_\omega, \theta\}$ tuples for perihelion and aphelion times from precomputed 2D interpolation functions, which turn out to be smooth and well-behaved in the domain of interest. One simulation, which is described in this poster, included 512 random-seeded trials for 2.5E8 orbits each, i.e., each was longer than 1 Gyr. It confirmed that 2006 HY51 could not come close to the rate of several thousand n required for rotational break-up.

The unexpected result was that all 512 simulations ended up in equilibrium states (resonances) within the simulation time span. Once a state of this kind was achieved, our fast tuple-generating simulation stopped behaving chaotically and repeated a certain combination of phase space parameters. Fig. 3 shows the aphelion spin of one such state with initial $\dot{\theta}$ / n = 966.514171, θ = 4.71115727 rad, confirmed by full-scale numerical integration.

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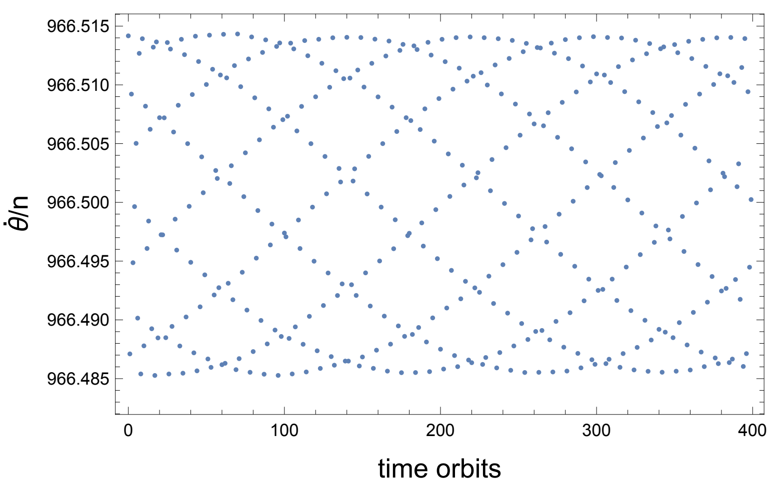

Fig.3 Aphelion spin rates of 2006 HY51 obtained by direct integration of 400 orbits in a state of regular (circulation) resonance.

The spin rate in this particular resonance displays a regular circulation behavior varying within a narrow range around $\dot{\theta}$ = 966.5 n. The aphelion orientation circulates about 0, i.e., the asteroid is aligned with the direction to the Sun. The width of variation depends on the initial perturbation but is limited to the width of the resonance. Fig. 4 shows the end-states of rotation when the asteroid is captured in a circulation resonance. Most of the resonances are above a certain threshold, which we pinpoint at 963.5, i.e., 1927:2 is the lowest possible resonance. There is a simple explanation why lower commensurability equilibria are unstable related to the range of velocity updates in Fig. 3. Only semi-integer commensurabilities are found for these high-spin circulation resonances, i.e., κ:1 and κ:2. 90% of our trials that ended in one of the regular resonances did this in less than 11E6 orbits, with a median duration of 3.4E6 orbits.

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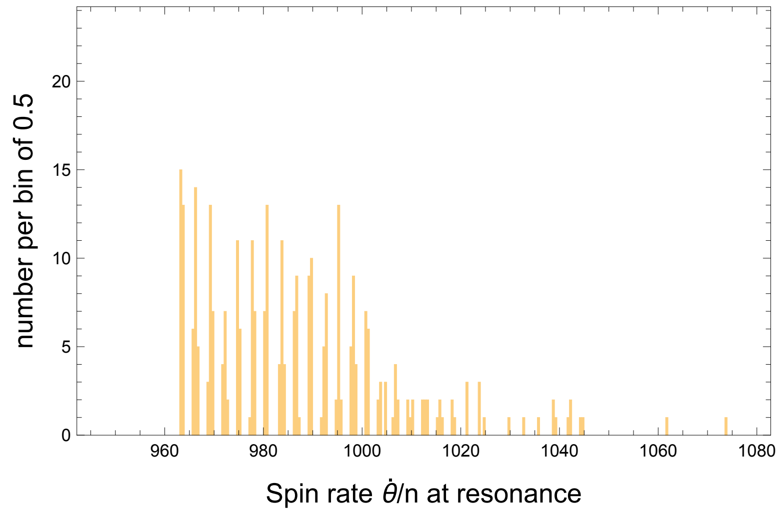

Fig.4 Histogram of end spin states of regular circulation resonances.

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Quantile[hy[[reso, 5]], {0.1, 0.2, 0.5, 0.8, 0.9}]
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Out[ ]= {661 847, 1 468 985, 3 405 471, 7 548 606, 10 935 858}

• Mapping a regular circulation resonance at high spin

Once we have identified approximate location of high spin resonances in the parameter space $\{\Theta_{aphelion}, \omega_{aphelion}\}$ using extensive simulations with the fast tuple-generation method, these tiny zones of equilibrium can be mapped in greater detail by direct numerical integration. Fig. 4 shows a parameter space cross section (similar to a Poincaré map) at aphelion obtained from 20 integration trials of 200 orbits each with random initial parameters in the vicinity of the 964:1 resonance. It shows the actual half-width of the circulation island, which is 0.21 rad in Θ and 0.17 n in ω.

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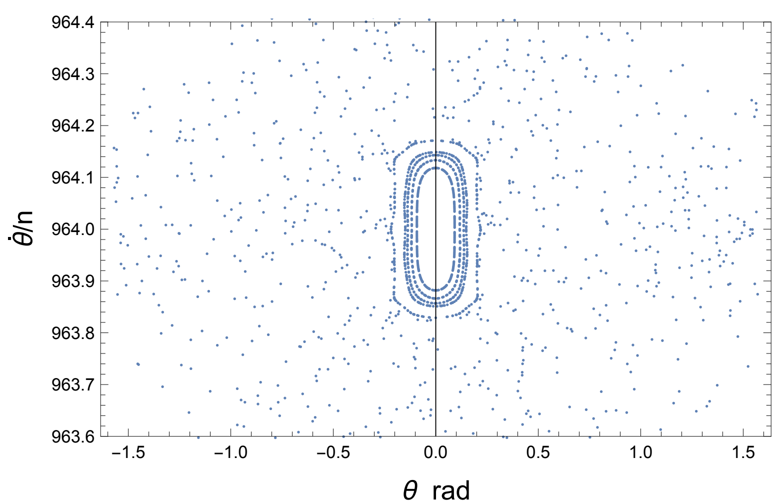

Fig.5 Aphelion parameter section of the 964:1 spin-orbit resonance of 2006 HY51.

This particular resonance has aphelion orientation angle close to 0 (or π), so that the asteroid is nearly aligned with the direction to the Sun. Multiple resonances are found with θ modulo π close to $\pi/2$, i.e., oriented sidewise at aphelia. This is not a new type of spin-orbit resonance since a sidewise capture into resonance has been discussed and deemed possible for the Moon, for example. A chain of such sidewise resonances is mapped in Fig. 6.

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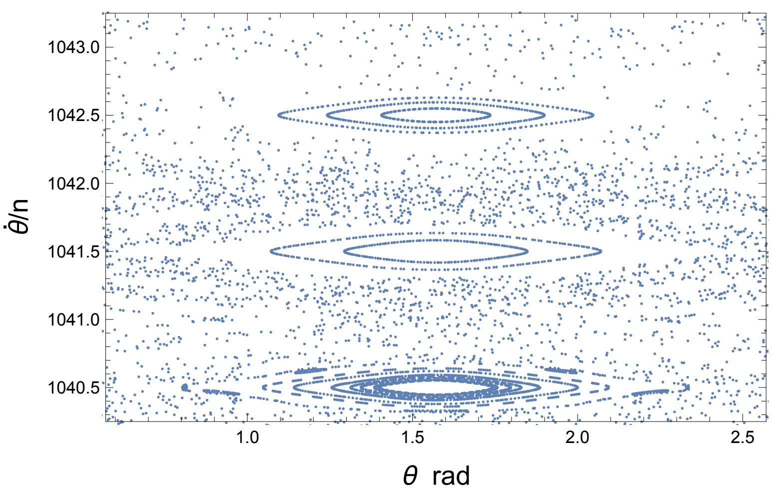

Fig.6 Aphelion parameter section of three adjacent spin-orbit resonance of 2006 HY51 with sidewise orientation of the asteroid.

• Three kinds of high spin resonances

Regular circulation resonances constitute 2/3 of our trials and are characterized by the same apoastron and periastron θ modulo π, which can be 0 (aligned) or $\pi/2$ (sidewise), and a stable, weakly librating rotation velocity. The remaining 1/3 of cases represent two new kinds of spin-orbit resonance, which may not have been described in the literature.

The first new kind, called *switching* resonance, is characterized by a nearly constant aphelion velocity ω, which is in a higher order of commensurability f with the mean motion n, and aphelion orientation cycling through integer multiples of π/f. One example is the resonance at ω = 1042.25 n (i.e., 4169:4, f = 4). The spin rate appears to chaotically vary within a very narrow range of 0.0006 n, while $\theta_{aphelion}$ modulo π switches between $\pi/4$ and $3\pi/4$ between each consecutive orbits, see Fig. 7. The aphelion tilt of 45 degrees compensates the fractional part of the relative spin, to the effect that the perihelion orientation angle switches between $\pi/2$ (sidewise) and π (aligned) in increments of integer multiple of $\pi/2$. Our simulations indicate that this resonance is rarely achieved through chaotic evolution, probably due to its narrowness.

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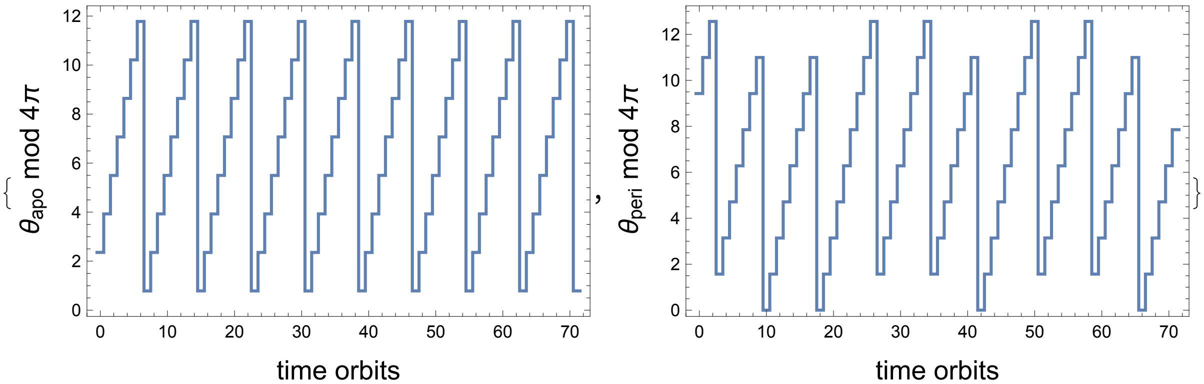

Fig.7 Aphelion (left) and perihelion (right) orientation angles modulo 4π in the 4169:4 switching spin-orbit resonance of 2006 HY51.

The second new kind, called *jumping* resonance, is characterized by a nearly constant aphelion orientation θ, which is a multiple of $\pi/2$ (sidewise), and aphelion velocity jumping between two quantum states for each pair of orbits, separated from an exact commensurability by a finite value. One example is the resonance at ω = 958.57923 n alternating with 958.42041 n, see Fig. 8. This non-commensurate resonance appears to be more rare than the regular circulation resonance possibly because of the narrow range in θ.

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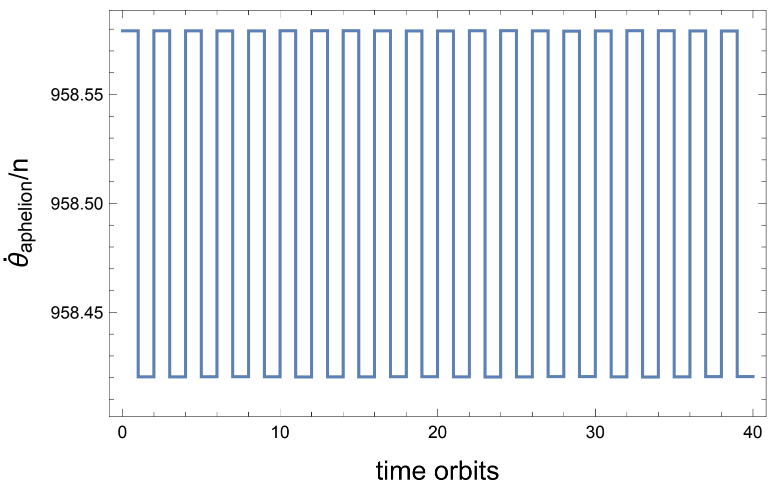

Fig.8 Aphelion rotation velocity in the (958.5±0.079)n jumping spin-orbit resonance of 2006 HY51.

• Conclusions and future work

- Simulating chaotic rotation of high-eccentricity asteroids with our fast tuple-generating method for ~1 Gyr time intervals, we discovered existence of high spin-orbit resonances
- These stable resonances may serve as protection against rotational break up at higher eccentricity, which needs to be investigated separately
- Apart from regular circulation resonances maintaining both aphelion velocity and orientation angle nearly constant, we discovered two new kinds of spin-orbit resonances: a switching resonance alternating the orientation angle, and a non-commensurate jumping resonance alternating the aphelion velocity between two quantum states
- The characteristic times of capture with 2006 HY51 parameters are below 100 Myr
- The 2006 HY51 may be captured in one of such resonances, which would be interesting to verify by observation
- Because of the narrowness of these equilibrium states, a moderate external perturbation (from an inner planet, for example) may extract the asteroid and trigger another chaotic walk for millions years
- Full 3D simulations of Euler’s equations will be needed to check the stability of high-spin resonances in the presence of obliquity wobble.