

Abstract: Orbital resonances are notoriously difficult to explain due to the complex dynamical processes that govern such orbital motion. One of the most famous examples is Pluto's 3:2 mean motion resonance with Neptune. A less well known aspect of Pluto's resonant orbital dynamics is a secular resonance constraining its perihelion drift in the third dimension. We describe a software tool to visualize the two and three dimensional aspects of Pluto's resonant orbital dynamics over time. This tool can be extended to visualize other orbital resonances.

Mean Motion Resonance (MMR)

- An orbital resonance between two bodies orbiting a common mass whose orbital periods are near an integer ratio p:q
- Examples: Kuiper Belt objects in resonance with Neptune, asteroids in resonance with Jupiter, planets in the exoplanetary system TRAPPIST-1
- The Neptune-Pluto 3:2 system is a famous example: Neptune completes 3 orbits while Pluto completes 2 orbits around the Sun in the same time interval

Challenges with MMR

- The effects of an MMR are notoriously difficult to explain due to the non-intuitive complex dynamics that govern such orbital motion
- A common technical approach is to identify a combination of Keplerian orbital angles to determine the resonant argument φ for a pair of objects: $\varphi = p\lambda_{m1} - q\lambda_{m2} - (p-q)\overline{\omega}$
- \circ If ϕ oscillates about a central value, the objects are said to be in MMR
- This technical approach is not readily translated into physical implications
- Also difficult to explain the differences between MMR and near-MMR cases
- Need a way to explain MMR in a physically relatable way that demonstrates the complex dynamics

Visualizing Pluto's Resonant Orbit in 4D Nihaal Zaveri¹ & Renu Malhotra²

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- This project: Visualize Pluto's orbit in 3 dimensions + time • We simulate the 3-body problem of Sun+Neptune+Pluto in Python using *Rebound* with the 'whfast' orbit integrator
- Orbital elements and masses for the Sun, Neptune and Pluto are imported from the JPL-Horizons database directly into Rebound Option to manually input Keplerian orbital elements and masses
- The 3-body system is integrated for 2e5 years, with outputs every 5 years, and the motion of the bodies is visualized in a rotating frame
- Using Neptune's mean angular velocity over 2e5 years, calculate the angle to Neptune's starting position at each timestep, rotate the system about the barycenter by this angle
- an animated movie of all the snapshots, in sequence



The figures above display the results of visualizing the system in the rotating frame. Panels 1-3 show the resonant orbit of Pluto as traced in the rotating frame in (x,y), (y,z), and (x,y,z) respectively. The figures for the Pluto-Neptune system were created using orbital elements and masses imported from JPL-Horizons. Panels 4-6 show an example of a non-MMR case visualized in the same rotating frame as panels 1-3. Panels 4-6 were created by manually inputting the orbital elements and masses for all 3 bodies, but Pluto's semimajor axis was increased by 1% to shift it out of MMR with Neptune. (Animations available at github.)

 \circ Create a snapshot of the (x,y,z) positions of all three objects at each timestep, then generate

References [Rebound] Rein and Tamayo (2015), A fast and unbiased implementation of a symplectic Wisdom-Holman integrator for long term gravitational simulations, Monthly Notices of the Royal Astronomical Society 452:376 [Pluto's resonant orbit dynamics] Malhotra and Williams (1997), Pluto's Heliocentric Orbit, in Pluto and Charon, Wiley-VCH, eds. Stern and Tholen



Analysis of Rotating Frame Plots

• In the rotating frame, we see that Neptune makes a small oval pattern at a nearly fixed distance from the Sun, while Pluto follows a librating orbital pattern (in a retrograde direction) around

the Sun • In the (x,y) projection, we see that:

- Pluto completes two "perihelion loops" in the rotating frame
- The "perihelion loops" librate around a position centered at 90° and 270° away from Neptune's longitude position
- The libration period is about 20,000 years • Pluto's MMR with Neptune is characterized by the oscillation of its "perihelion loops" about central values of ± 90 degrees from Neptune's longitude
- In the non-MMR case, the loops instead continue to circulate all the way around
- In the (x,z) and (y,z) and (x,y,z) projections:
- We see that Pluto reaches its lowest z-position at apocenter and highest z-position at pericenter
- This is related to the vZ-L-K (vonZeipel-Lidov-Kozai) oscillation

Future Work

- Python code will be updated to handle more than 3 bodies at once; next step is to include all 4 giant planets and recreate these visualizations in a more realistic model of the Solar system
- Further explore vZ-L-K dynamics of Pluto
- Apply to other resonant Kuiper Belt objects,
- Asteroids and extrasolar systems
- Software tools will be available on GitHub https://github.com/renumalhotra/2021-Pluto-s-Re sonant-Dynamics-Visualized-in-4D.git